COHOMOLOGY OF TANGO BUNDLE ON $\mathbb{P}^5$

DANIELE FAENZI

Abstract. The Tango bundle T over $\mathbb{P}^5$ is proved to be the pull–back of the twisted Cayley bundle $C(1)$ via a map $f\colon \mathbb{P}^5 \to Q_5$ existing only in characteristic 2. The Frobenius morphism φ factorizes via such f.

1. The bundles on $\mathbb{P}^5$ and on Q_5

The well–known Hartshorne conjecture states, in particular, that there are no indecomposable rank-2 vector bundles on $\mathbb{P}^n$, when n is greater than 5. However, one of the few rank-2 bundles on $\mathbb{P}^5$ up to twist and pull–back by finite morphisms is the Tango bundle T first given in [Tan76]. Later Horrocks in [Hor78] and Decker Manolache and Schreyer in [DMS92] discovered that it can be obtained starting from Horrocks rank-3 bundle. In [Fae03] we proved that T is the pull–back of the Cayley bundle over the quadric Q_5. Anyway it only exists in characteristic 2.

Here we compute the cohomology of T applying an analogue of Borel–Bott–Weil theorem in positive characteristic. In section (1) we introduce the involved bundles and state the theorem, while in section (2) we give proofs and some more remarks. We will need to compute the cohomology of $\varphi^*(C)$, $\mathrm{Sym}^2(C)$ and C, where φ is the degree 32 Frobenius morphism $\varphi : \mathbb{P}^5 \to \mathbb{P}^5$, defined when $\mathrm{char}(k) = 2$. We make use of `Macaulay2` computer algebra package, see [GS].

Let k be an algebraically closed field, and let Q_5 be the smooth 5-dimensional quadric hypersurface over k. On the coordinate ring $R(\mathbb{P}^5)$ we use variables x_i's while on $R(Q_5)$ we use z_j's.

$$Q_5 = \{z_0^2 + z_1 z_2 + z_3 z_4 + z_5 z_6 = 0\} \subset \mathbb{P}^6$$

Both on $\mathbb{P}^5$ and Q_5, we denote Chern classes by integers, meaning integral multiples of the positive generators in each degree of the Chow ring. Further, we call *intermediate cohomology group* any sheaf cohomology group of degree other than 0 or 5, i.e. the dimension of the ambient variety.

On $Q_5 = \mathsf{G}_2/\mathsf{P}(\alpha_1)$ we have the Cayley bundle C, associated to the standard representation of the semisimple part of the parabolic group $\mathsf{P}(\alpha_1)$,

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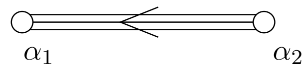
where α_1 is the shortest root in the Lie algebra of the exceptional Lie group G_2, having the following Dynkin diagram.



The bundle C is obtained as the irreducible $\mathsf{P}(\alpha_1)$-homogeneous module with maximal weight $\lambda_2 - 2\lambda_1$. The bundle $C(2)$ (weight λ_2) is globally generated and we have $\mathrm{h}^0(C(2)) = 14$. By virtue of [Fae03, Theorem (2)] we can define the Tango bundle as follows.

Definition. Let $\mathrm{char}(k) = 2$. For any odd n there exists a non–constant morphism $f\colon \mathbb{P}^n \longrightarrow Q_n$. For $n = 5$ the pull–back $T = f^*(C(1))$ is a stable rank-2 bundle with $c_1(T) = 2$, $c_2(T) = 4$ and Hilbert polynomial:

$$\chi(T(t)) = \frac{1}{60}t^5 + \frac{1}{3}t^4 + \frac{25}{12}t^3 + \frac{11}{3}t^2 - \frac{51}{10}t - 14$$

Therefore we have, for every $0 \leq p \leq 5$ and $t \in \mathbb{Z}$:

$$T \simeq T^*(2) \qquad \text{and} \qquad \mathrm{h}^p(T(t)) = \mathrm{h}^{5-p}(T(-t-8))$$

Theorem 1. *Given the Tango bundle T defined above, we have:*

$$\mathrm{h}^0(T(t)) = \chi(T(t)) \qquad \textit{for } t \geq 3$$

Furthermore, the only nonvanishing intermediate cohomology of T is the following:

t	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2
$\mathrm{h}^1(T(t))$									1	7	14	13	1
$\mathrm{h}^2(T(t))$								1					
$\mathrm{h}^3(T(t))$						1							
$\mathrm{h}^4(T(t))$	1	13	14	7	1								

In particular, the bundle $T' = T(-4)$ is the only twist of T for which all cohomology groups vanish.

Recall the definition of the spinor 4-bundle S over Q_5. Since $Q_5 = \mathsf{Spin}(7)/\mathsf{P}(\beta_1)$, where β_1 is the shortest root of $\mathsf{Spin}(7)$, we define S as the bundle associated to the spin representation of the semisimple part $ss(\mathsf{P}(\beta_1)) = \mathsf{Spin}(5)$. The bundle $S^\vee = S(1)$ is globally generated. By Beilinson's theorem, the bundle $S(1)$, extended by zero to $\mathbb{P}^6$, has the minimal graded free resolution:

$$0 \longrightarrow \mathcal{O}_{\mathbb{P}^6}(-1)^8 \xrightarrow{B} \mathcal{O}_{\mathbb{P}^6}^8 \longrightarrow S(1) \longrightarrow 0$$

where now B, by the observations in [Bea00], is an antisymmetric matrix whose determinant is the equation of the quadric, to the power 4. This is done by the matrix:

$$
B = \begin{pmatrix}
0 & 0 & 0 & -z_3 & 0 & -z_1 & z_5 & -z_0 \\
0 & 0 & z_3 & 0 & z_1 & 0 & -z_0 & -z_6 \\
0 & -z_3 & 0 & 0 & -z_5 & z_0 & 0 & -z_2 \\
z_3 & 0 & 0 & 0 & z_0 & z_6 & z_2 & 0 \\
0 & -z_1 & z_5 & -z_0 & 0 & 0 & 0 & z_4 \\
z_1 & 0 & -z_0 & -z_6 & 0 & 0 & -z_4 & 0 \\
-z_5 & z_0 & 0 & -z_2 & 0 & z_4 & 0 & 0 \\
z_0 & z_6 & z_2 & 0 & -z_4 & 0 & 0 & 0
\end{pmatrix} \tag{1}
$$

The bundle C is related to S in the following way. One computes $c_4(S^\vee) = 0$. So, given a nontrivial global section of $S^\vee$, we have a rank-3 bundle G defined by:

$$0 \longrightarrow \mathcal{O} \xrightarrow{a} S^\vee \longrightarrow G \longrightarrow 0 \tag{2}$$

It turns out that $c_3(G^\vee(1)) = 0$. One can prove that $G^\vee(1)$ has a unique section b and that the quotient by such b is isomorphic to $C(1)$ i.e. C is the cohomology of the monad:

$$\mathcal{O}(-1) \xrightarrow{b(-1)} S \xrightarrow{a^t} \mathcal{O} \tag{3}$$

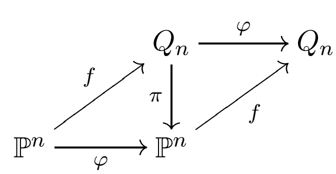
The bundle C has rank 2 and Chern classes $(-1, 1)$. The only non–vanishing intermediate cohomology groups are $\mathrm{H}^1(C) = \mathrm{H}^4(C(-4)) = k$. All this is done in [Ott90] and follows easily from [Jan87, Proposition 5.4] in any characteristic.

2. Proof of the theorem and further remarks

First of all we need some properties of the map $f\colon \mathbb{P}^n \longrightarrow Q_n$. Recall that it is defined for $\mathrm{char}(k) = 2$ and for any odd n as:

$$f(x_0 : \ldots : x_n) = (x_0x_1 + \cdots + x_{n-1}x_n : x_0^2 : \ldots : x_n^2)$$

The map f is a finite morphism of degree 2^{n-1} and factors the Frobenius morphism φ as in the diagram:



where π the projection from $(1 : 0 : \ldots : 0)$.

Observation 2. Given the map f defined above, we have:

$$
\begin{aligned}
f_*(\mathcal{O}_{\mathbb{P}^5}) &= \mathcal{O} \oplus \mathcal{O}(-1)^{14} \oplus \mathcal{O}(-2) \\
f_*(\mathcal{O}_{\mathbb{P}^5}(1)) &= S \oplus \mathcal{O}^6 \oplus \mathcal{O}(-1)^6
\end{aligned} \tag{4}
$$

Proof. Let $\mathcal{F} = f_*(\mathcal{O}_{\mathbb{P}^5}), \mathcal{G} = f_*(\mathcal{O}_{\mathbb{P}^5}(1))$. The map f is a $16\colon 1$ cover, because the Frobenius is $32\colon 1$ and the projection π is $2\colon 1$. Then $\mathcal{F}$ and $\mathcal{G}$ are rank-16 vector bundles, whose cohomology one can read from the Leray degenerate spectral sequence. Indeed since $R^i(f_*) = 0$ (for $i > 0$) we have:

$$\begin{aligned} &\mathrm{H}^i(Q_5, \mathcal{F}(t)) = \mathrm{H}^i(\mathbb{P}^5, f^*(\mathcal{O}_{Q_5}(t)) = \mathrm{H}^i(\mathbb{P}^5, \mathcal{O}_{\mathbb{P}^5}(2t)) \\ &\mathrm{H}^i(Q_5, \mathcal{G}(t)) = \mathrm{H}^i(\mathbb{P}^5, f^*(\mathcal{O}_{Q_5}(1)(t)) = \mathrm{H}^i(\mathbb{P}^5, \mathcal{O}_{\mathbb{P}^5}(2t+1)) \end{aligned} \tag{5}$$

for $0 \le i \le 5$ and every t. This says that $\mathcal{F}$ and $\mathcal{G}$ have no intermediate cohomology, hence by [Kap86] or [BGS87] they must decompose as sum of twisted spinor bundles $S(a)$ and line bundles (although actually Kapranov's setting is over $\mathbb{C}$). For $\mathcal{G}$ this implies, by a computation on the Euler characteristic, that the only choice is the one stated. On the other hand for $\mathcal{F}$ we have a priori two possibilities: either the one stated above either $S \oplus S(-1) \oplus \mathcal{O} \oplus \mathcal{O}(-1)^6 \oplus \mathcal{O}(-2)$.

Now the formula (5) says that the polynomial ring $R(\mathbb{P}^5)$ decomposes as a module over $R(Q_5)$ (under the action given by f) as $R(\mathbb{P}^5)_{\mathrm{even}} \oplus R(\mathbb{P}^5)_{\mathrm{odd}}$ where:

$$R(\mathbb{P}^5)_{\mathrm{even}} = \bigoplus_{t\in\mathbb{Z}} \mathrm{H}^0(\mathbb{P}^5, \mathcal{O}(2t)) \qquad R(\mathbb{P}^5)_{\mathrm{odd}} = \bigoplus_{t\in\mathbb{Z}} \mathrm{H}^0(\mathbb{P}^5, \mathcal{O}(2t+1))$$

For $\mathcal{F}$ we have to compute explicitly a presentation of the $R(Q_5)-$module $R(\mathbb{P}^5)_{\mathrm{even}}$. We need e_0 to generate $k = R(\mathbb{P}^5)_0$ and e_{ij} to generate the monomial $x_i x_j$ $(i \neq j)$ in $R(\mathbb{P}^5)_2$, thus obtaining a map:

$$\Phi\colon R(Q_5) \oplus R(Q_5)(-1)^{15} \longrightarrow R(\mathbb{P}^5)_0 \oplus R(\mathbb{P}^5)_2$$

But the coordinate e_{45} is redundant, since $z_0\Phi(e_0) + \Phi(e_{01}) + \Phi(e_{23}) = \Phi(e_{45})$. Now for $R(\mathbb{P}^5)_4$: the terms containing x_i^2 already lie in the image (got by the action of z_{i-1}), and in fact we just have to fix $x_0x_1x_2x_3$ because, e.g. $x_0x_1x_2x_4 = z_3\Phi(e_{34}) + z_5\Phi(e_{25}) + z_0\Phi(e_{24})$ and $x_0x_1x_4x_5 = x_0x_1x_2x_3 + z_1z_2\Phi(e_0) + z_0\Phi(e_{01})$. Thus we get a generator in degree 2 (and no syzygy); moreover $x_0x_1x_2x_3x_4x_5 = z_1z_2\Phi(e_{23}) + z_3z_4\Phi(e_{01}) + z_0(x_0x_1x_2x_3)$, so that $R(\mathbb{P}^5)_6$, and in fact all $R(\mathbb{P}^5)_{\mathrm{even}}$, is also covered.

The presentation can be in fact computed also for $R(\mathbb{P}^5)_{\mathrm{odd}}$. In this case one finds as syzygy of a map $R(Q_5)^6 \oplus R(Q_5)(-1)^{14} \to R(\mathbb{P}^5)_{\mathrm{odd}}$ the matrix B described in (1) giving the spinor bundle, thus getting again (4). □

Remark 3. Observation (2) can probably be extended to any odd dimension. Here we only mention that for $f\colon \mathbb{P}^3 \longrightarrow Q_3$ we get $f_*(\mathcal{O}_{\mathbb{P}^3}) = S \oplus \mathcal{O}_{Q_3} \oplus \mathcal{O}_{Q_3}(-1)$ and $f_*(\mathcal{O}_{\mathbb{P}^3}(1)) = \mathcal{O}_{Q_3}^4$ which can be computed by the presentation or by Euler characteristic.

Finally, one may notice that $\pi_*(S) = \mathcal{O}_{\mathbb{P}^5}(-1)^8$ and clearly $\pi_*(\mathcal{O}_{\mathbb{P}^5}(t)) = \mathcal{O}(t) \oplus \mathcal{O}(t-1)$ so that the extension $0 \to S(-1) \to \mathcal{O}(-1)^8 \to S \to 0$ splits after π_* (actually this holds in any characteristic). This agrees with the formula:

$$\varphi_*(\mathcal{O}_{\mathbb{P}^5}) = \mathcal{O} \oplus \mathcal{O}(-1)^{15} \oplus \mathcal{O}(-2)^{15} \oplus \mathcal{O}(-3)$$
$$\varphi_*(\mathcal{O}_{\mathbb{P}^5}(1)) = \mathcal{O}^6 \oplus \mathcal{O}(-1)^{20} \oplus \mathcal{O}(-2)^6$$

Next we compute the cohomology of $\operatorname{Sym}^2 C$, $C \otimes C$, $S \otimes C$. First notice that if V is the $\mathsf{SL}(2)$–representation giving C, when $\operatorname{char}(k) = 2$ the representation $\operatorname{Sym}^2 V$ (having weight $2\lambda_2 - 4\lambda_1$) will not be irreducible (recall that in finite characteristic $\mathsf{SL}(2)$ is not linearly reductive, check [Nag62]). On the contrary, letting $C^{[2]} = \varphi^*(C)$, we have the non–split exact sequence:

$$0 \longrightarrow C^{[2]} \longrightarrow \operatorname{Sym}^2 C \longrightarrow \mathcal{O}(-1) \longrightarrow 0 \tag{6}$$

We also have:

$$\begin{aligned} \varphi_*(\mathcal{O}_{Q_5}) &= \mathcal{O} \oplus \mathcal{O}(-1)^{20} \oplus \mathcal{O}(-2)^7 \oplus S(-1) \\ \varphi_*(\mathcal{O}_{Q_5}(1)) &= \mathcal{O}^7 \oplus \mathcal{O}(-1)^{20} \oplus \mathcal{O}(-2) \oplus S \end{aligned} \tag{7}$$

Now again by Borel–Bott–Weil theorem ([Jan87, Proposition 5.4]) we know $\mathrm{h}^i(\operatorname{Sym}^2 C(3)) = 0$ for all i, $\operatorname{Sym}^2 C(4)$ is globally generated and:

$$\mathrm{h}^0(\operatorname{Sym}(t)) = \chi(\operatorname{Sym}^2 C(t)) = \frac{1}{20}t^5 + \frac{3}{8}t^4 - \frac{27}{8}t^2 - \frac{81}{20}t \quad \text{for } t \geq 4$$
$$\mathrm{h}^1(\operatorname{Sym}^2 C(2)) = 14 \quad \mathrm{h}^1(\operatorname{Sym}^2 C(1)) = 7 \quad \mathrm{h}^1(\operatorname{Sym}^2 C(-1)) = 1$$

where in the above twists these are the only non–vanishing H^i's. By Serre duality we only miss $\mathrm{H}^2(\operatorname{Sym}^2 C)$. Now since $R^i(\varphi_*) = 0$, by the degenerate Leray spectral sequence, (6) and (7) we get:

$$\begin{aligned} \mathrm{H}^2(\operatorname{Sym}^2 C) =& \mathrm{H}^2(C^{[2]}) \overset{\text{Leray}}{=} \mathrm{H}^2(C \otimes S(-1)) \overset{\text{Leray}}{=} \\ &= \mathrm{H}^2(C^{[2]}(-1)) = \mathrm{H}^2(\operatorname{Sym}^2 C(-1)) \overset{\text{Bott}}{=} k \end{aligned}$$

and by the same argument $\mathrm{h}^1(\operatorname{Sym}^2 C) = 1$, while the remaining h^i are zero. The same procedure yields the following values of $\mathrm{h}^i(S \otimes C(t))$:

$$\begin{aligned} &\mathrm{h}^1(S \otimes C) = 1 && \mathrm{h}^1(S \otimes C(1)) = 6 \\ &\mathrm{h}^2(S \otimes C(-1)) = 1 && \mathrm{h}^3(S \otimes C(-2)) = 1 \\ &\mathrm{h}^4(S \otimes C(-4)) = 6 && \mathrm{h}^4(S \otimes C(-5)) = 1 \end{aligned} \tag{8}$$

Now we can prove Theorem 1.

Proof of Theorem 1. Since $R^i(f_*) = 0$ for $i > 0$ we compute cohomology over Q_5 and get:

$$\begin{aligned} \mathrm{H}^i(\mathbb{P}^5, T(2t)) =& \mathrm{H}^i(Q_5, f_*(\mathcal{O}_{\mathbb{P}^5}) \otimes C(1+t)) \\ \mathrm{H}^i(\mathbb{P}^5, T(2t+1)) =& \mathrm{H}^i(Q_5, f_*(\mathcal{O}_{\mathbb{P}^5}(1)) \otimes C(1+t)) \end{aligned} \tag{9}$$

The even part gives $\mathrm{H}^i(\mathbb{P}^5, T(2t)) = \mathrm{H}^i(Q_5, C(1+t)) \oplus \mathrm{H}^i(Q_5, C(t))^{14} \oplus \mathrm{H}^i(Q_5, C(t-1))$, while the odd part gives $\mathrm{H}^i(\mathbb{P}^5, T(2t+1)) = \mathrm{H}^i(Q_5, C(1+$

$t))^6 \oplus \mathrm{H}^i(Q_5, C(t))^6 \oplus \mathrm{H}^i(Q_5, S \otimes C(1+t))$ and these are known by the above formulas. □

Remark 4. The values $\mathrm{h}^2(\mathrm{Sym}^2 C) = \mathrm{h}^1(\mathrm{Sym}^2 C) = 1$ exhibit non–standard cohomology for the representation $\mathrm{Sym}^2 V$. Indeed $3\lambda_2 - 3\lambda_1$ is singular ($(3\lambda_2 - 3\lambda_1, 3\alpha_1 + \alpha_2) = 0$) so standard Borel–Bott–Weil theorem (i.e. in characteristic 0) would give $\mathrm{h}^i(\mathrm{Sym}^2 C) = 0$. Of course, we would have no such sequence as (6).

Still, by tensoring the monad defining C by $C(t)$ we get:

$$0 \longrightarrow G^\vee \otimes C(t) \longrightarrow S \otimes C(t) \longrightarrow C(t) \longrightarrow 0$$
$$0 \longrightarrow C(t-1) \longrightarrow G^\vee \otimes C(t) \longrightarrow C \otimes C(t) \longrightarrow 0$$

whence we derive the values of $\mathrm{h}^i(S \otimes C(t))$ from those of $\mathrm{Sym}^2 C$, because, if $\mathrm{char}(k) \neq 2$, $C \otimes C = \mathrm{Sym}^2 C \oplus \mathcal{O}(-1)$. This way we would get the same values as in (8) if $\mathrm{char}(k) = 0$. Finally, one can compute on `Macaulay2` the values of the cohomology of $C^{[2]}$ and check the correctness of the above result.

Now we can write the Beilinson table of the normalized $T(-1)$:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Then we write $T(-1)$ as cohomology of a monad:

$$\mathcal{O}(-1) \oplus \Omega^4(4) \longrightarrow \Omega^2(2) \oplus \Omega^1(1) \longrightarrow \mathcal{O}^7$$

As Wolfram Decker pointed out to us, another way to get Tango's bundle is by Horrocks bundle $\mathcal{H}$ in characteristic 2. Concretely, Horrocks becomes a non–split extension $0 \to T(-1) \to \mathcal{H} \to \mathcal{O} \to 0$. This allows to compute the cohomology of $\mathcal{H}$ in terms of T.

Moreover in [DMS92] one finds an explicit description of the maps in the Beilinson monad. This provides also a check of computations. Denoting with $e_0, \dots, e_5$ the canonical basis of E (the exterior algebra over $W = \mathrm{H}^0(\mathbb{P}^5, \mathcal{O}(1))$), and using the natural isomorphism $\mathrm{Hom}(\Omega^i(i), \Omega^j(j)) =$

$\wedge^{i-j} W = E_{j-i}$, T is the cohomology of the maps α and β:

$$\beta = \begin{pmatrix} e_0 & e_4e_5 \\ e_1 & e_3e_5 \\ e_2 & e_3e_4 \\ e_3 & e_1e_2 \\ e_4 & e_0e_2 \\ e_5 & e_0e_1 \\ 0 & e_0e_3 + e_1e_4 + e_2e_5 \end{pmatrix}$$

$$\alpha = \begin{pmatrix} e_0e_1e_2 + e_3e_4e_5 & e_0e_1e_3e_4 + e_0e_2e_3e_5 + e_1e_2e_4e_5 & 0 \\ e_0e_3 + e_1e_4 + e_2e_5 & e_0e_1e_2 + e_3e_4e_5 & e_1e_2e_4e_5 \end{pmatrix}$$

Finally, using the equations for T, `Macaulay2` provides the following resolution:

$$0 \longrightarrow \begin{matrix} \mathcal{O}^7(-7) \\ \oplus \\ \mathcal{O}(-8) \end{matrix} \longrightarrow \mathcal{O}^{49}(-6) \longrightarrow \mathcal{O}^{98}(-5) \longrightarrow \mathcal{O}^{76}(-4) \longrightarrow \begin{matrix} \mathcal{O}^7(-3) \\ \oplus \\ \mathcal{O}^{14}(-2) \end{matrix} \longrightarrow T \longrightarrow 0$$

Applying cohomology algorithms in `Macaulay2` developed by Decker Eisenbud and Schreyer one may also obtain a full table of the cohomology, which we write in `Macaulay2` notation:

(10)

total:	573	260	92	27	14	7	2	2	7	14	27	92	260	573
-6:	573	260	91	14	.	.	.	.	.	.	.	.	.	.
-5:	.	.	1	13	14	7	1	.	.	.	.	.	.	.
-4:	.	.	.	.	.	.	1	.	.	.	.	.	.	.
-3:	.	.	.	.	.	.	.	1	.	.	.	.	.	.
-2:	.	.	.	.	.	.	.	1	7	14	13	1	.	.
-1:	.	.	.	.	.	.	.	.	.	.	14	91	260	573

Reading the table along one antidiagonal gives the list of cohomology groups of a single twist. Here the list for T starts from the up–right corner, while starting from a shift to the left means reading the list for a (-1)–twist. Table (10) agrees with Theorem 1.

References

[Bea00] Arnaud Beauville, *Determinantal hypersurfaces*, Michigan Math. J. **48** (2000), 39–64, Dedicated to William Fulton on the occasion of his 60th birthday. MR 2002b:14060

[BGS87] Ragnar-Olaf Buchweitz, Gert-Martin Greuel, and Frank-Olaf Schreyer, *Cohen-Macaulay modules on hypersurface singularities. II*, Invent. Math. **88** (1987), no. 1, 165–182. MR 88d:14005

[DMS92] Wolfram Decker, Nicolae Manolache, and Frank-Olaf Schreyer, *Geometry of the Horrocks bundle on* $\mathbf{P}^5$, Complex projective geometry (Trieste, 1989/Bergen, 1989), London Math. Soc. Lecture Note Ser., vol. 179, Cambridge Univ. Press, Cambridge, 1992, pp. 128–148. MR 94h:14043

[Fae03] Daniele Faenzi, *A Geometric Construction of Tango Bundle on* $\mathbb{P}^5$, Kodai Mathematical Journal 27 (2004), no. 1, 1–6.

[GS] Daniel R. Grayson and Michael E. Stillman, *Macaulay 2, a software system for research in algebraic geometry*, Available at http://www.math.uiuc.edu/Macaulay2/.

[Hor78] Geoffrey Horrocks, *Examples of rank three vector bundles on five-dimensional projective space*, J. London Math. Soc. (2) **18** (1978), no. 1, 15–27. MR 80d:14011

[Jan87] Jens Carsten Jantzen, *Representations of algebraic groups*, Pure and Applied Mathematics, vol. 131, Academic Press Inc., Boston, MA, 1987. MR 89c:20001

[Kap86] Mikhail M. Kapranov, *Derived category of coherent bundles on a quadric*, Funktsional. Anal. i Prilozhen. **20** (1986), no. 2, 67. MR 88a:14014

[Nag62] Masayoshi Nagata, *Complete reducibility of rational representations of a matric group.*, J. Math. Kyoto Univ. **1** (1961/1962), 87–99. MR 26 #236

[Ott90] Giorgio Ottaviani, *On Cayley bundles on the five-dimensional quadric*, Boll. Un. Mat. Ital. A (7) **4** (1990), no. 1, 87–100. MR 91c:14055

[Tan76] Hiroshi Tango, *On morphisms from projective space P^n to the Grassmann variety $Gr(n,d)$*, J. Math. Kyoto Univ. **16** (1976), no. 1, 201–207. MR 53 #5614

Daniele Faenzi
Dipartimento di Matematica U. Dini
Università di Firenze
Viale Morgagni 67/a
I-50134, Florence, Italy
e-mail: faenzi@math.unifi.it
home page: http://www.math.unifi.it/faenzi/

E-mail address: faenzi@math.unifi.it

Dipartimento di Matematica "U. Dini", Università di Firenze, Viale Morgagni 67/a, I-50134, Florence, Italy

URL: http://www.math.unifi.it/~faenzi/